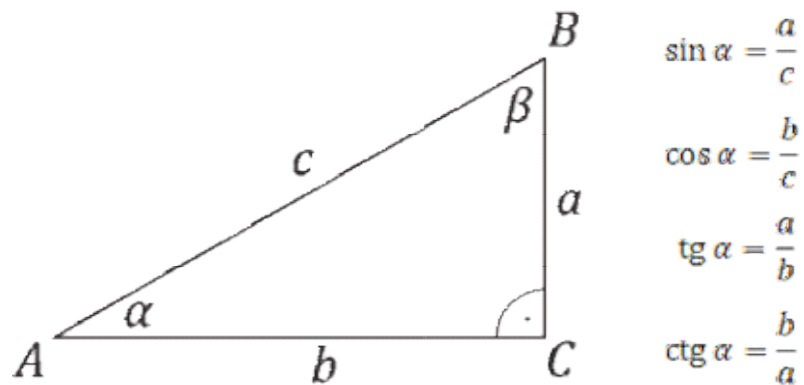




Materijal

https://www.ffh.bg.ac.rs/uploads/sr/2020/10/PMFH_skripta_trig-ostar-ugao.pdf

Неке вредности тригонометријских функција

α	0°	30°	45°	60°	90°	180°	270°	360°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin \alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos \alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
$\operatorname{tg} \alpha$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	∞	0
$\operatorname{ctg} \alpha$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	∞	0	∞
$\sec \alpha$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	-1	∞	1
$\operatorname{cosec} \alpha$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	∞	-1	∞

Identiteti samo nam treba za sinus i kosinus:

Trigonometrijske formule

- **Osnovne** formule

$$\sin^2 x + \cos^2 x = 1$$

$$\pi = 180^\circ$$

$$\operatorname{tg} x = \frac{\sin x}{\cos x}; x \neq \frac{\pi}{2} + \pi k$$

$$k \in \mathbb{Z}$$

$$\operatorname{ctg} x = \frac{\cos x}{\sin x}; x \neq \pi k$$

$$\operatorname{tg} x = \frac{1}{\operatorname{ctg} x}$$

$$\sin(360^\circ k + x) = \sin x$$

$$\cos(360^\circ k + x) = \cos x$$

$$\operatorname{tg}(180^\circ k + x) = \operatorname{tg} x$$

$$\operatorname{ctg}(180^\circ k + x) = \operatorname{ctg} x$$

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\operatorname{tg}(-x) = -\operatorname{tg} x$$

$$\operatorname{ctg}(-x) = -\operatorname{ctg} x$$

- **Adicione** formule

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\operatorname{tg} x + \operatorname{tg} y$$

video:

https://www.youtube.com/watch?v=GPhPQwl_QQE

Trigonometrijski krug u Geogebri:

<https://www.geogebra.org/m/A7XPqxdM>

Zadaci iz zbirke

$$1.) \text{ Ako } \tan x = \frac{5}{13} \text{ odrediti } \tan(2x) \cdot \left(0 \leq x \leq \frac{\pi}{2} \right)$$

rešenje:

$$\tan(2x) = \frac{\sin(2x)}{\cos(2x)} = \frac{2 \sin x \cos x}{\cos^2 x - \sin^2 x}$$

$$\cos x = ?$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x = 1 - \sin^2 x$$

$$\cos^2 x = 1 - \left(\frac{5}{13}\right)^2 = 1 - \frac{25}{169} = \frac{144}{169}$$

$$\cos x = \pm \sqrt{\frac{144}{169}} = \pm \frac{12}{13}$$

$$0 \leq x \leq \frac{\pi}{2} \rightarrow \cos x \geq 0 \rightarrow$$

$$\cos x = \frac{12}{13}$$

$$\tan(2x) = \frac{2 \sin x \cos x}{\cos^2 x - \sin^2 x} = \frac{2 \cdot \frac{5}{13} \cdot \frac{12}{13}}{\frac{144}{169} - \frac{25}{169}} = \frac{120}{119}$$

2.) ІЗРАЇНАТИ

$$\sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{6} - \frac{\pi}{4}\right) =$$

$$\sin\frac{\pi}{3} \cos\frac{\pi}{4} + \cos\frac{\pi}{3} \sin\frac{\pi}{4} =$$

$$\left(\cos\frac{\pi}{6} \cos\frac{\pi}{4} - \sin\frac{\pi}{6} \sin\frac{\pi}{4}\right) =$$

$$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \left(\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}\right)$$

$$\boxed{\frac{\sqrt{2}}{2}}$$

3) ПОКЕДАТИ ОД НАТМАНЕ ДО
НАЈВЕЌЕ ВРЕДНОСТИ

$$\sin\left(\frac{2022\pi}{3}\right), \operatorname{tg}\left(\frac{800\pi}{6}\right), \cos\left(\frac{5\pi}{5}\right);$$
$$\operatorname{ctg}\left(\frac{10^6\pi}{3}\right)$$

$$\sin\left(\frac{2022\pi}{3}\right) = \sin(2 \cdot 1011\pi)$$
$$= \sin(337 \cdot (2\pi)) = \sin(0) = 0$$

$$\operatorname{tg}\left(\frac{800\pi}{6}\right) = \operatorname{tg}\left(133\pi + \frac{2\pi}{3}\right) = \operatorname{tg}\left(\frac{\pi}{3}\right) = \sqrt{3}$$

$$\cos\left(\frac{5\pi}{5}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$$

$$\operatorname{ctg}\left(\frac{10^6\pi}{3}\right) = \operatorname{ctg}\left(333333\pi + \frac{\pi}{3}\right) =$$
$$\operatorname{ctg}\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{3}$$

Резултат: $\sin\left(\frac{2022\pi}{3}\right), \cos\left(\frac{5\pi}{5}\right),$
 $\operatorname{ctg}\left(\frac{10^6\pi}{3}\right), \operatorname{tg}\left(\frac{800\pi}{6}\right)$