**1.** Rešiti jednačinu (u skupu realnih brojeva): $(x^4 - 8x^2 - 20)(x^3 - 1) = 0$.

РЕШЕЊЕ:

$x^4 - 8x^2 - 20 = 0$

СМЕНА: $t = x^2$, $t^2 = x^4$

$t^2 - 8t - 20 = 0$

$(t - 10)\cdot(t+2) = 0$

$\boxed{t_1 = 10}$ $\boxed{t_2 = -2}$

$t_1$: $x^2 = 10 \Rightarrow x = \pm\sqrt{10}$

$\boxed{x_1 = -\sqrt{10}}$ $\boxed{x_2 = \sqrt{10}}$

$t_2$: $x^2 = -2$ НЕМА РЕАЛНИХ РЕШЕЊА

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$x^3 - 1 = 0$ $\quad x^3 = 1 \Rightarrow x = 1$

$(x-1)(x^2+x+1) = 0$

$x - 1 = 0 \Rightarrow \boxed{x_3 = 1}$

$x^2 + x + 1 = 0$

$x_{4/5} = \dfrac{-1 \pm \sqrt{1-4}}{2} = \dfrac{-1 \pm \sqrt{-3}}{2}$

НЕМА РЕАЛНИХ РЕШЕЊА

**7.** Naći $x$ iz jednačine (u skupu realnih brojeva): $x^6 - 3x^3 + 2 = 0$.

РЕШЕЊЕ:

СМЕНА! $t = x^3$, $t^2 = x^6$

$$t^2 - 3t + 2 = 0$$

$$(t-1)(t-2) = 0$$

$$\boxed{t_1 = 1} \quad \boxed{t_2 = 2}$$

$$t_1: x^3 = 1 \Rightarrow \boxed{x_1 = 1}$$

$$t_2: x^3 = 2$$

$$x^3 - 2 = 0$$

$$x^3 - \left(\sqrt[3]{2}\right)^3 = 0$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$\left(x - \sqrt[3]{2}\right)\left(x^2 + \sqrt[3]{2}\,x + \sqrt[3]{4}\right) = 0$$

$$x - \sqrt[3]{2} = 0 \Rightarrow \boxed{x_2 = \sqrt[3]{2}}$$

$x^2 + \sqrt[3]{2}\,x + \sqrt[3]{4} = 0$ НЕМА РЕАЛНИХ РЕШЕЊА

**9.** Rešiti jednačinu (u skupu realnih brojeva): $(x^2 + x)^2 - 8(x^2 + x) + 12 = 0$

<u>РЕШЕЊЕ!</u>

СМЕНА: $t = x^2 + x$

$t^2 - 8t + 12 = 0$

$(t-2)\cdot(t-6) = 0$

$\boxed{t_1 = 2}, \boxed{t_2 = 6}$

$t_1$: $x^2 + x = 2$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$\boxed{x_1 = -2}$ $\boxed{x_2 = 1}$

$t_2$: $x^2 + x = 6$

$x^2 + x - 6 = 0$

$(x+3)\cdot(x-2) = 0$

$\boxed{x_3 = -3}$ $\boxed{x_4 = 2}$

**1.** Izračunati: $\dfrac{2-3i}{3+2i}+\dfrac{3+2i}{2-3i}$

$$\boxed{i^2=-1}$$

<u>РЕШЕЊЕ:</u>

$$\frac{(2-3i)\cdot(2-3i)+(3+2i)\cdot(3+2i)}{(3+2i)\cdot(2-3i)}$$

$$=\frac{\cancel{4}-\cancel{6i}-\cancel{6i}-\cancel{9}+\cancel{9}+\cancel{12}i-\cancel{4}}{6-9i+4i+6}$$

$$=\frac{0}{12-5i}=\boxed{0}$$

**4.** Izračunati: $(3 + 2i)^5$

<u>РЕШЕЊЕ:</u>

$$(3+2i)\cdot\left((3+2i)^2\right)^2$$

$$= (3+2i)\cdot(9+12i-4)^2 = (3+2i)(5+12i)^2$$

$$= (3+2i)\cdot(25+120i-144)$$

$$= (3+2i)(-119+120i)$$

$$= -357+360i-238i-240$$

$$= \boxed{-597+122i}$$

**6.** Rešiti jednačinu: $x^4 + 2x^2 + 1 = 0.$

РЕШЕЊЕ:

СМЕНА: $t = x^2$, $t^2 = x^4$

$t^2 + 2t + 1 = 0$

$(t+1)^2 = 0$

$\boxed{t_1 = t_2 = -1}$

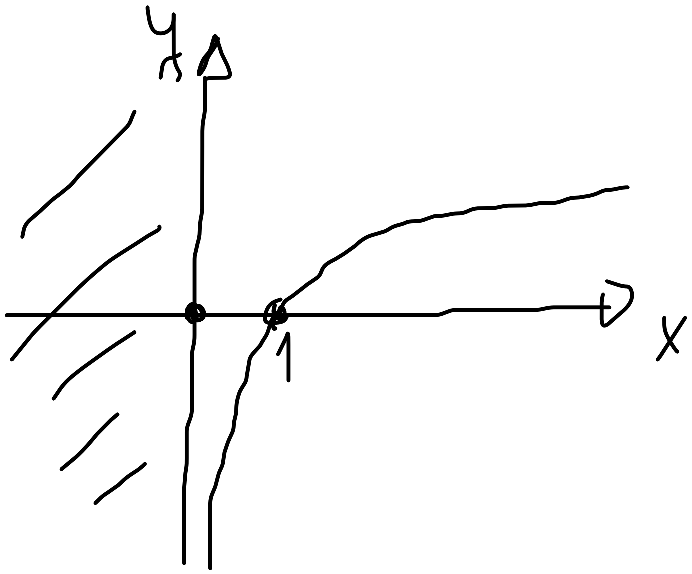
$t_1$: $x^2 = -1$

$\boxed{x_1 = -i}$ $\boxed{x_2 = i}$

$t_2$: $x^2 = -1$

$\boxed{x_3 = -i}$ $\boxed{x_4 = i}$

# Логаритамска функција




$$f(x) = \log_a x$$

$$x > 0 \;!$$

$$a = e, \quad f(x) = \ln x$$

$$\ln 1 = 0$$

$$a = 10, \quad f(x) = \log b$$

$$\log_a 1 = 0$$

$$\log_a b = c \iff a^c = b$$

$$n \cdot \log_a b = \log_a b^n$$

$$\log_{a^n} b = \frac{1}{n} \log_a b$$

$$\log_a b + \log_a c = \log_a (b \cdot c)$$

**1.** Rešiti logaritamsku jednačinu: $2(\log x)^2 + \log x - 1 = 0.$

РЕШЕЊЕ:

СМЕНА: $t = \log x$

$2t^2 + t - 1 = 0$

$t_{1/2} = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4}$

$\boxed{t_1 = \frac{1}{2}}$ $\boxed{t_2 = -1}$

$t_1: \log x = \frac{1}{2} \Rightarrow \boxed{x_1 = 10^{\frac{1}{2}} = \sqrt{10}}$

$t_2: \log x = -1 \Rightarrow \boxed{x_2 = 10^{-1} = \frac{1}{10}}$

**14.** Rešiti jednačinu: $\log_5(x+1) + \log_5(x-3) = 1$.

<u>РЕШЕЊЕ:</u>

$\log_5 (x+1)(x-3) = 1$

$(x+1)(x-3) = 5^1$

$x^2 - 3x + x - 3 = 5$

$x^2 - 2x - 8 = 0$

$(x-4)\cdot(x+2) = 0$

$\boxed{x_1 = 4}$ $\boxed{\cancel{x_2 = -2}}$

Решити једначину: $\log_2(x+1)+\log_4(x+7)^2=4$.

**Решење:**

$$\log_2(x+1)+\log_{2^2}(x+7)^2=4$$

$$\log_2(x+1)+\frac{1}{\cancel{2}}\cdot\cancel{2}\cdot\log_2(x+7)=4$$

$$\log_2(x+1)(x+7)=4$$

$$x^2+7x+x+7=2^4$$

$$x^2+8x-9=0$$

$$(x+9)\cdot(x-1)=0$$

$$\cancel{x_1=-9} \qquad \boxed{x_2=1}$$

Решити једначину: $\underbrace{\log\sqrt{x-5}+\log\sqrt{2x-3}}+1=\log 30.$

$1=\log 10$

Решење:

$$\log\left(\sqrt{x-5}\cdot\sqrt{2x-3}\right)=\log 30-\log 10$$

$$\log\sqrt{(x-5)(2x-3)}=\log\frac{30}{10}$$

$$\sqrt{2x^2-3x-10x+15}=3 \quad {}^2$$

$$2x^2-13x+15=9$$

$$2x^2-13x+6=0$$

$$x_{1/2}=\frac{13\pm\sqrt{169-48}}{4}=\frac{13\pm 11}{4}$$

$$\boxed{x_1=6}$$

$$\boxed{\cancel{x_2=\frac{1}{2}}}$$

# Степена ф-ја

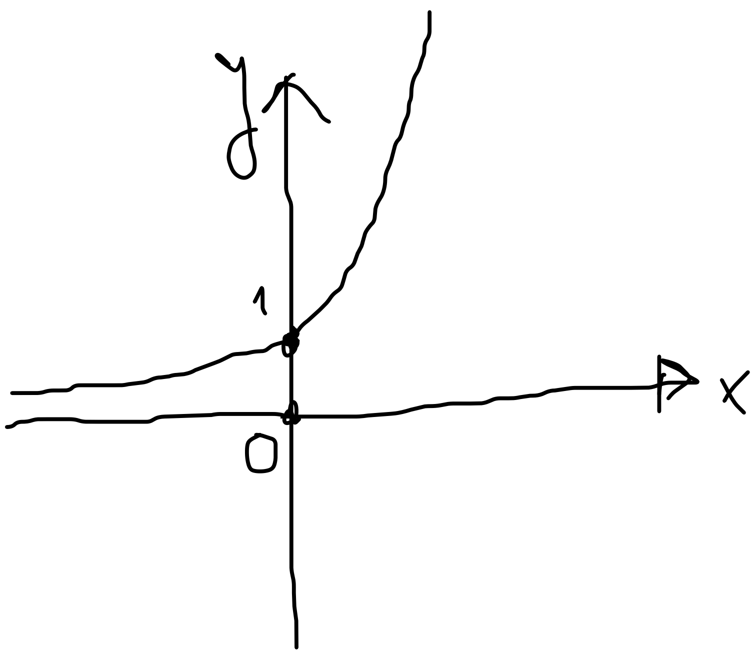
$$\boxed{a^{-n} = \frac{1}{a^n}}$$

$$\boxed{x^{\frac{a}{b}} = \sqrt[b]{x^a}}$$

$$\boxed{x^a \cdot x^b = x^{a+b}}$$

$$\boxed{\frac{x^a}{x^b} = x^{a-b}}$$

$$\boxed{(x^a)^b = x^{a \cdot b}}$$




$$f(x) = e^x$$

$$\boxed{e^0 = 1} \qquad \boxed{e^1 = e}$$

**4.** Odrediti $x$ iz eksponencijalne jednačine: $3^{2x} - 10 \cdot 3^x + 9 = 0.$

<u>РЕШЕЊЕ:</u>

<u>СМЕНА:</u> $t = 3^x$ $\quad t^2 = (3^x)^2 = 3^{2x}$

$t^2 - 10t + 9 = 0$

$(t-1)(t-9) = 0$

$\boxed{t_1 = 1}$ $\quad \boxed{t_2 = 9}$

$t_1$: $3^{x} = 1 = 3^{0} \Rightarrow \boxed{x_1 = 0}$

$t_2$: $3^x = 9 = 3^2 \Rightarrow \boxed{x_2 = 2}$

**7.** Naći rešenja jednačine: $7^{5x} = 3.$

РЕШЕЊЕ!

$$\boxed{a^c = b \Leftrightarrow \log_a b = c}$$

$$5x = \log_7 3$$

$$x = \frac{1}{5}\log_7 3$$

$$\boxed{x = \log_7 \sqrt[5]{3}}$$

**10.** Naći $x$ iz jednačine: $$e^{2x^2} \cdot e^{5x^2} = 7$$

Решење:

$$\boxed{\ln(e^x) = x}$$

$$e^{2x^2+5x^2} = 7$$

$$e^{7x^2} = 7 \quad / \ln$$

$$7x^2 = \ln 7$$

$$x^2 = \frac{1}{7} \ln 7 \qquad x^2 = \ln \sqrt[7]{7}$$

$$\boxed{x_1 = -\sqrt{\ln \sqrt[7]{7}}} \qquad \boxed{x_2 = \sqrt{\ln \sqrt[7]{7}}}$$

**16.** Rešiti eksponencijalnu jednačinu: $2^{x+2} + 2^x = 40$.

<u>РЕШЕЊЕ:</u>

$$2^x \cdot 2^2 + 2^x = 40$$

$$4 \cdot 2^x + 2^x = 40$$

$$5 \cdot 2^x = 40$$

$$2^x = 8 = 2^3 \Rightarrow \boxed{x = 3}$$

Решити једначину: $(\sqrt{3})^{x^2-x}=27$.

Решење:

$$\left(3^{\frac{1}{2}}\right)^{x^2-x}=3^3$$

$$3^{\frac{1}{2}(x^2-x)}=3^3$$

$$\frac{1}{2}(x^2-x)=3$$

$$x^2-x=6$$

$$x^2-x-6=0$$

$$(x-3)\cdot(x+2)=0$$

$$\boxed{x_1=3} \quad \boxed{x_2=-2}$$

## АРИТМЕТИЧКИ НИЗ

$$a_1, a_2, a_3, \ldots, a_n$$

$$a_1 \xrightarrow{+d} a_2 \xrightarrow{+d} a_3 \quad \cdots \quad \xrightarrow{+d} a_n$$

$$\boxed{a_n = a_1 + (n-1)\cdot d}$$

$$\boxed{S_n = \sum_{i=1}^{n} a_i = \frac{a_1 + a_n}{2}\cdot n}$$

## ГЕОМЕТРИЈСКИ НИЗ

$$b_1, b_2, b_3, \ldots, b_n$$

$$b_1 \xrightarrow{\cdot q} b_2 \xrightarrow{\cdot q} b_3 \quad \cdots \quad \xrightarrow{\cdot q} b_n$$

$$\boxed{b_n = b_1 \cdot q^{n-1}}$$

$$\boxed{S_n = b_1 \cdot \frac{q^n - 1}{q - 1}}$$

**3.** Naći sumu aritmetičkog niza: $3 + 6 + 9 + \dots + 93.$

$+3 \quad +3$

<u>РЕШЕЊЕ:</u>

$a_1 = 3$

$a_n = 93$

$n = 31$

$d = 3$

$$n - 1 = \frac{a_n - a_1}{d} = \frac{93 - 3}{3} = 30$$

$$\boxed{n = 31}$$

$$S_n = \frac{a_1 + a_n}{2} \cdot n = \frac{3 + 93}{2} \cdot 31$$

$$\boxed{S_n = 48 \cdot 31 = 1488}$$

**8.** Odrediti sumu niza: $\overbrace{1 + \underbrace{2^2 + 2^3 + \ldots + 2^{50}}_{S_n}}^{S}$.

РЕШЕЊЕ:

$1, \; 4, \; 8, \; 16, \; \ldots, \; 2^{50}$

$\cdot 4 \quad \cdot 2 \quad \cdot 2 \quad \cdot 2$

$b_1 = 4$

$q = 2$

$n = 49$

$$S_n = b_1 \cdot \frac{q^n - 1}{q - 1} = 4 \cdot \frac{2^{49} - 1}{2 - 1} = 2^{51} - 4$$

$$\boxed{S = 1 + S_n = 2^{51} - 3}$$

**9.** Izračunati sumu niza: $\underbrace{4 + 6 + 8 + \ldots + 168 + 170}$ i naći 41. član tog niza.

+2 +2 +2

РЕШЕЊЕ:

$a_1 = 4$

$a_n = 170$

$d = 2$

$$S_n = \frac{a_1 + a_n}{2} \cdot n = \frac{4 + 170}{2} \cdot 84 = 174 \cdot 42$$

$$\boxed{S_n = 7308}$$

$$a_{41} = a_1 + (41 - 1) \cdot d$$

$$\boxed{a_{41} = 4 + 40 \cdot 2 = 84}$$

$$n - 1 = \frac{a_n - a_1}{d} = \frac{170 - 4}{2} = \frac{166}{2}$$

$$\boxed{n = 84}$$

**10.** Odrediti sumu: $\overbrace{3+6+12+\ldots+96+192}^{S_1}+\overbrace{375+750+1500+3000}^{S_2}.$

$\cdot 2 \quad \cdot 2 \quad \cdot 2 \quad \cdot 2 \quad \cdot 2 \quad \cdot 2$

<u>Решење:</u>

$$\boxed{S=}\, S_1+S_2 = 381+5625=\boxed{6006}$$

$b_1 = 3$

$b_n = 192$

$q = 2$

$b_n = b_1 \cdot q^{n-1}$

$192 = 3\cdot 2^{n-1}$

$2^{n-1} = 64 = 2^6 \Rightarrow n-1=6,\ \boxed{n=7}$

$$S_1 = b_1 \cdot \frac{q^n-1}{q-1} = 3\cdot\frac{2^7-1}{2-1} = 3\cdot 127 = \boxed{381}$$

$S_2$: $b_1 = 375,\ q=2,\ n=4$

$$S_2 = 375\cdot\frac{2^4-1}{2-1} = 375\cdot 15 = \boxed{5625}$$