Kvadratna funkcija

$$y = ax^2 + bx + c,\ a \neq 0.$$

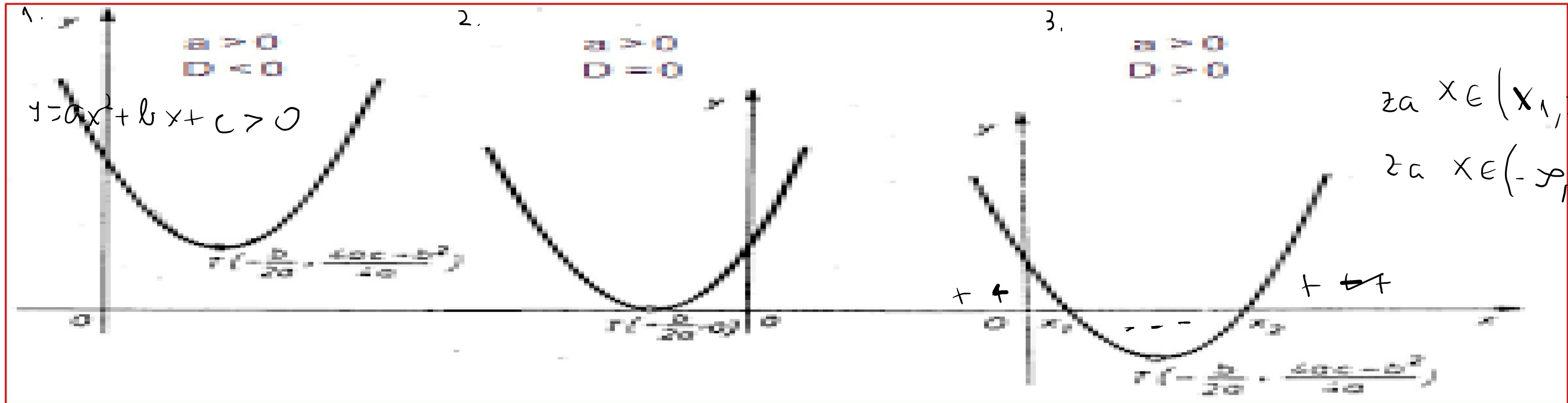
Teme: $T\left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right)$

$$D = b^2 - 4ac$$

Grafik:

$$a > 0$$

$$x_{1/2} = \frac{-b \pm \sqrt{\underbrace{b^2-4ac}_{D}}}{2a}$$



za $x \in (x_1, x_2)$ $y < 0$

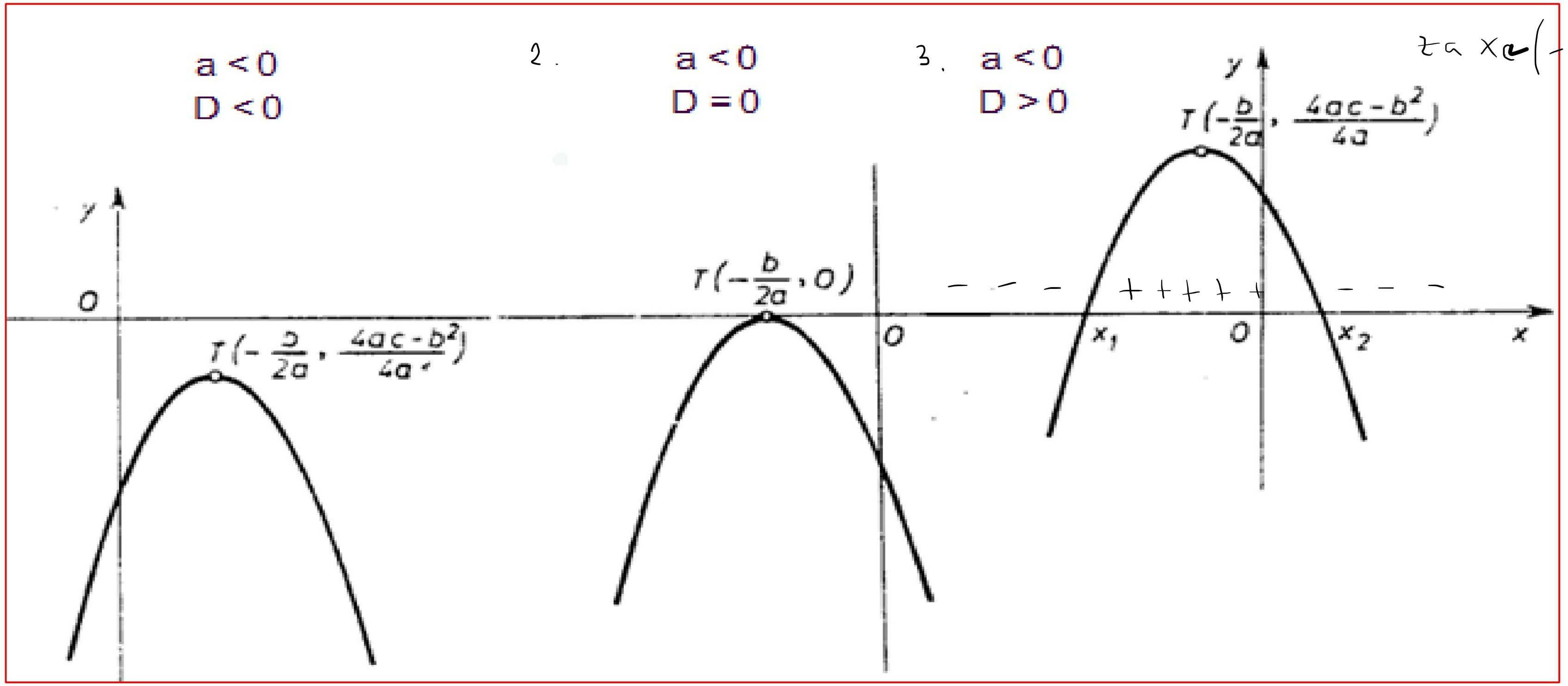
za $x \in (-\infty, x_1) \cup (x_2, \infty)$ $y > 0$

$$a < 0$$

za $x \in (x_1, x_2)$ $y > 0$

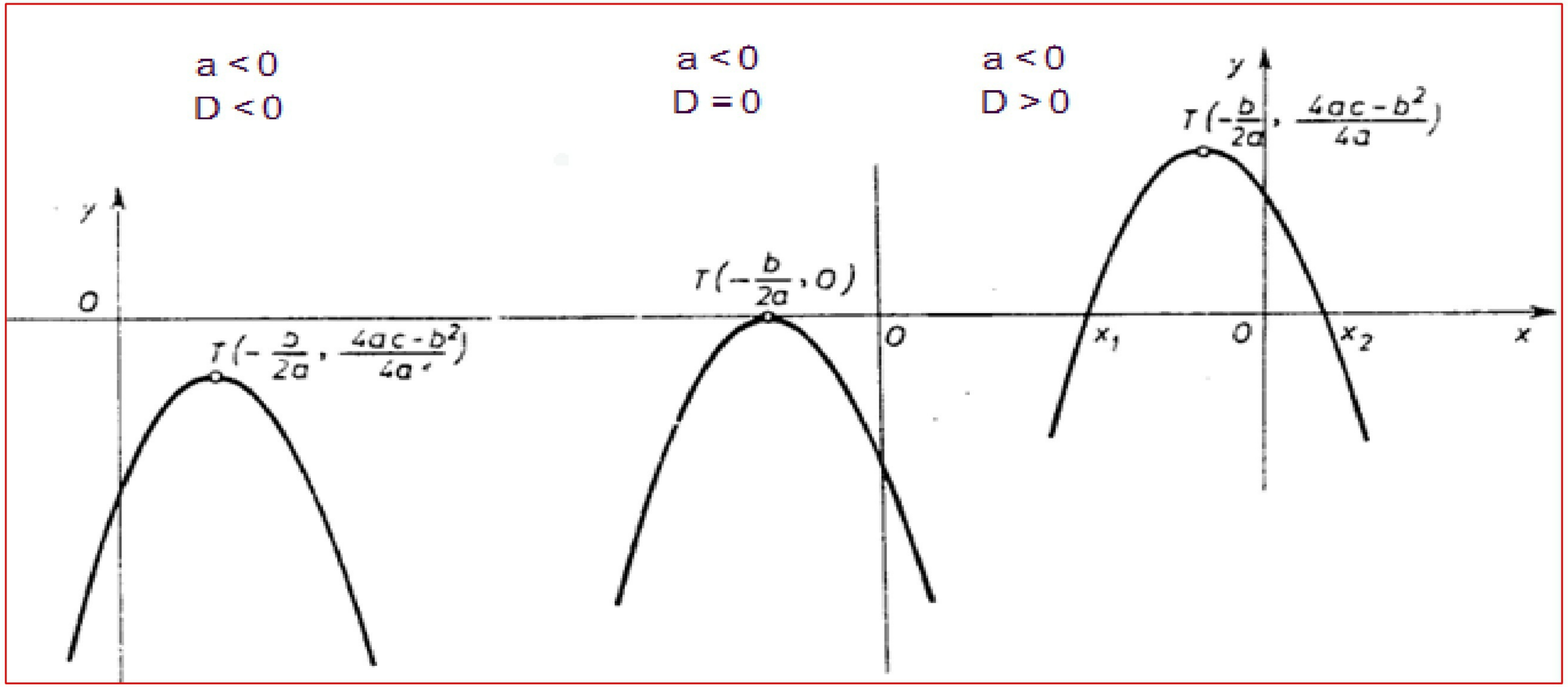
za $x \in (-\infty, x_1) \cup (x_2, \infty)$ $y < 0$

1.



$y < 0$

$a < 0$



1. Odrediti teme i skicirati grafik parabole $y = x^2 + ax + b$ ako seče y–osu u $A(\underset{x}{0}, \underset{y}{-4})$ i sadrži tačku $B(\underset{x}{2}, \underset{y}{-6})$.

$$y = x^2 + ax + b$$

$$-4 = 0^2 + a\cdot 0 + b \Rightarrow b = -4$$

$$-6 = 2^2 + 2a + b$$

$$-6 = \cancel{4} + 2a - \cancel{4}$$

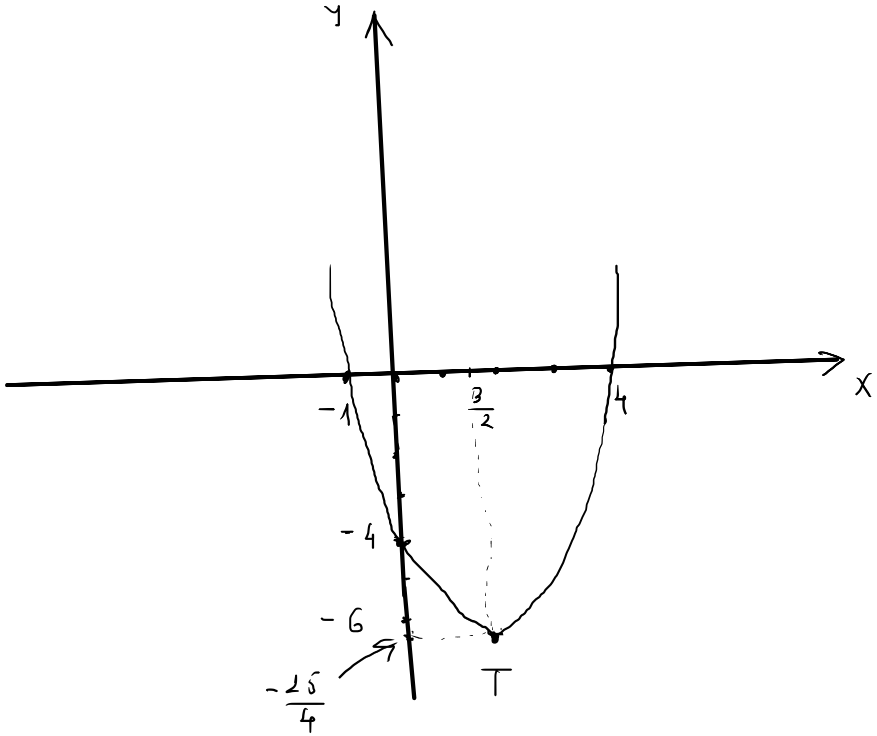
$$a = \frac{-6}{2} = -3$$

$$\boxed{y = x^2 - 3x - 4} \qquad y = ax^2 + bx + c$$

$$T\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$$

$$T\left(+\frac{3}{2}, \frac{4\cdot(-4) - (-3)^2}{4}\right) = T\left(\frac{3}{2}, \frac{-25}{4}\right)$$

$$x_{1/2} = \frac{3 \pm \sqrt{9 + 16}}{2} = \frac{3 \pm 5}{2} \qquad x_1 = 4 \quad x_2 = -1$$



2. Odrediti parametre a i c za funkciju $y = ax^2 + 2x + c$ ako funkcija ima maksimum u tački $A(1,1)$ i grafički predstaviti krivu. Za koje x je $y > 0$ a za koje je $y < 0$ i odrediti vrednost funkcije u tački $x = 3$.

$y = ax^2 + 2x + c \qquad a = a, \; b = 2, \; c = c$

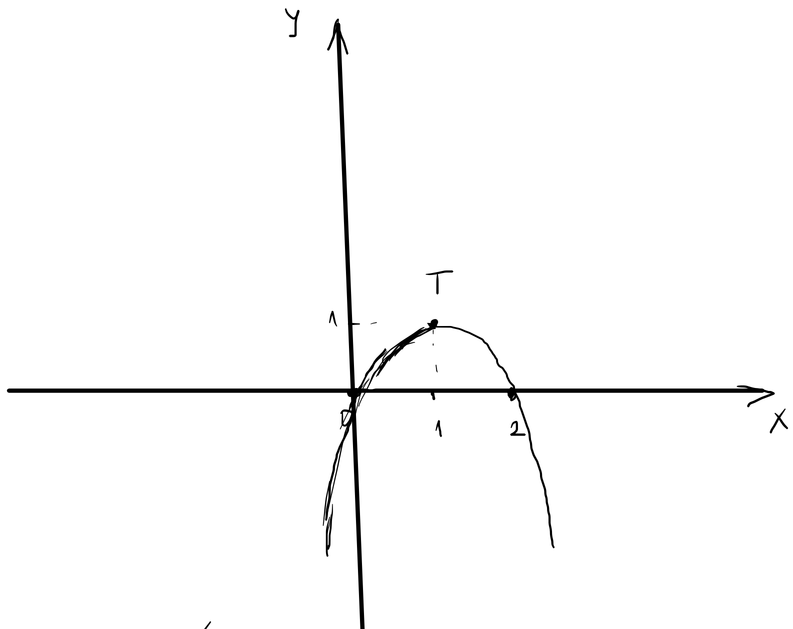
$T\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right) = T(1,1)$

$-\frac{2}{2a} = 1 \Rightarrow a = -1$

$\frac{-4c - 2^2}{-4} = 1 \Rightarrow -4c - \cancel{4} = -\cancel{4} \Rightarrow c = 0$

$y = -x^2 + 2x = x(-x+2)$

$y = 0$ za $x_1 = 0$ ili $x_2 = 2$



za $x \in (0,2)$ $y > 0$

za $x \in (-\infty, 0) \cup (2, \infty)$ $y < 0$

za $x = 3$ $\quad y = -3^2 + 2 \cdot 3 = -3$

3. Odrediti a tako da $y = x^2 + ax - 6$ seče x—osu u $x = 2$, odrediti presek i grafički predstaviti krivu.

$$0 = 2^2 + 2a - 6$$

$$0 = 2a - 2$$

$$2a = 2$$

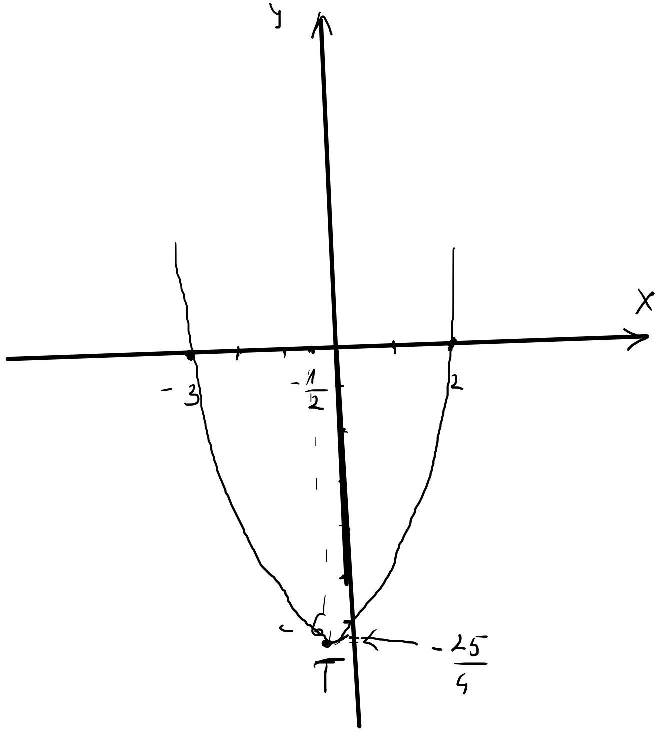
$$a = 1$$

$$\boxed{y = x^2 + x - 6}$$

$$y = (x-2)(x+3) \quad \text{ili} \quad x_{1/2} = \frac{-1 \pm \sqrt{1+24}}{2} = \frac{-1 \pm 5}{2}$$

$$x_1 = 2 \qquad x_2 = -3$$

$$T\left(-\frac{1}{2}, \frac{4\cdot(-6) - 1^2}{4}\right) = T\left(-\frac{1}{2}, -\frac{25}{4}\right)$$



4. Odrediti a, b i c u $y = ax^2 + bx + c$ ako je teme $T(-1, 1)$ i grafik seče y–osu u $y = 2$. Grafički predstaviti krivu. Da li je T maksimum ili minimum? Izračunati vrednost funkcije u $x = -2$.

$y = ax^2 + bx + c$, $a \neq 0$

$(0,2)$ je tačka preseka na y-osom.

$2 = a \cdot 0 + b \cdot 0 + c \Rightarrow c = 2$

$T\left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right) = T(-1,1)$

$\frac{-b}{2a} = -1 \Rightarrow -b = -2a \quad b = 2a$

$\frac{8a-b^2}{4a} = 1 \quad \frac{8a-(2a)^2}{4a} = 1 \quad 8a - 4a^2 = 4a$

$4a - 4a^2 = 0$

$\cancel{4a}(1-a) = 0$

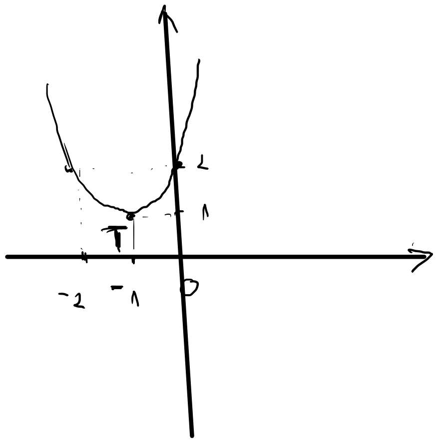
$\cancel{a=0}$ jer y nije kvadratna f-ja

$a = 1 \quad b = 2 \cdot 1 = 2$

$\boxed{y = x^2 + 2x + 2}$

T je minimum

$x_{1/2} = \frac{-2 \pm \sqrt{4-8}}{2} \in \mathbb{C}$



$y(-2) = (-2)^2 + 2 \cdot (-2) + 2 = 2$

5. Odrediti b i c u $y = x^2 + bx + c$ ako seče x—osu u $x = 1$ i $x = -3$, grafički predstaviti krivu i odrediti krivu.

$y = x^2 + bx + c$

$0 = 1 + b + c$

$0 = (-3)^2 - 3b + c$

$b + c = -1 \quad / \cdot (-1)$

$-3b + c = -9$

$-4b = -8$

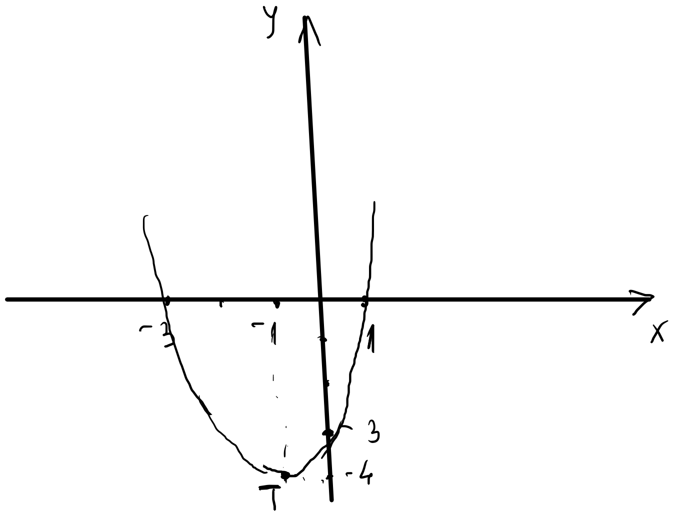
$b = 2$

$2 + c = -1 \Rightarrow c = -3$

$y = x^2 + 2x - 3$

$x_1 = 1, \ x_2 = -3$ nule f-je, tj. preseci na x-osom

$T\left(\frac{-2}{2}, \frac{4 \cdot (-3) - 2^2}{4}\right) = T(-1, -4)$



6. Odrediti parametar a tako da je funkcija $y = x^2 - 4ax + a^4$ dodiruje $x-$osu i grafički predstaviti.

$y = x^2 - 4ax + a^4 \Rightarrow \underbrace{a = 1 \quad b = -4a \quad c = a^4}$

$D = 0$

$D = b^2 - 4ac = (-4a)^2 - 4 \cdot 1 \cdot a^4 = 16a^2 - 4a^4 = 4a^2(4 - a^2) = 0$

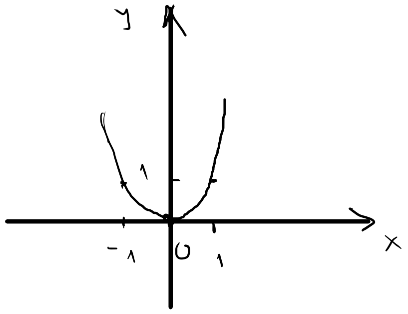
$a_1 = 0 \qquad 4 - a^2 = 0$

$(2-a)(2+a) = 0$

$a_2 = 2 \qquad a_3 = -2$

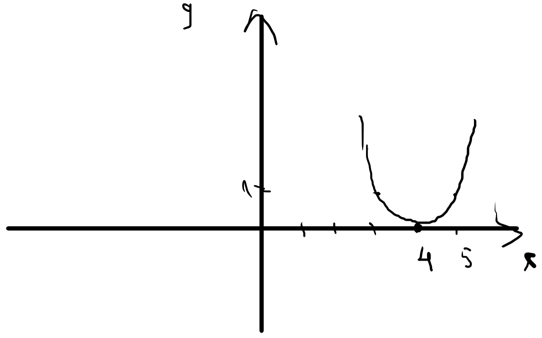
• $a_1 = 0$

$y = x^2$



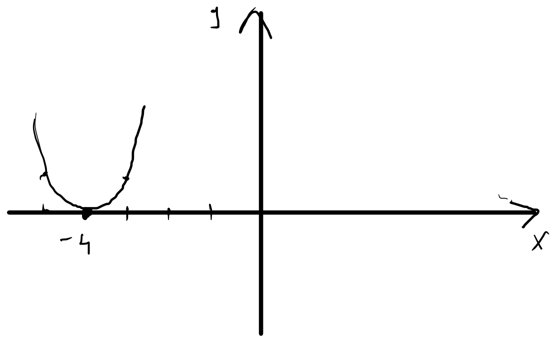
• $a_2 = 2$

$y = x^2 - 8x + 16 = (x-4)^2$



• $a_3 = -2$

$y = x^2 + 8x + 16 = (x+4)^2$

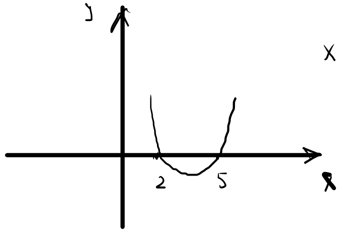


Nejednacine

(1.) $x^2-7x+10>0$

I način: $x_{1/2}=\frac{7\pm\sqrt{49-40}}{2}=\frac{7\pm3}{2}$

$x_1=5 \quad x_2=2$



$x\in(-\infty,2)\cup(5,\infty)$

II način: $x_1=5 \quad x_2=2$

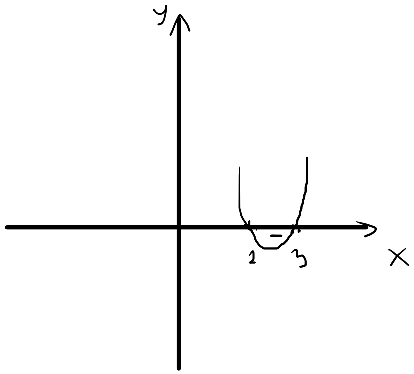
$(x-5)\cdot(x-2)>0$

	$(-\infty,2)$	2	$(2,5)$	5	$(5,\infty)$
$x-5$	$-$		$-$	0	$+$
$x-2$	$-$	0	$+$		$+$
$(x-5)\cdot(x-2)$	$+$	0	$-$	0	$+$

$x\in(-\infty,2)\cup(5,\infty)$

(2.) $x^2-5x+6\leq 0$

$x_{1/2}=\frac{5\pm\sqrt{25-24}}{2}=\frac{5\pm1}{2} \qquad x_1=3 \qquad x_2=2$



$x\in[2,3]$

3.

$$\frac{5x+8}{4x-3} > 1$$

$$\frac{5x+8}{4x-3} - 1 > 0 \qquad |\cdot(4x-3)$$

$$\frac{5x+8-4x+3}{4x-3} > 0$$

$$\frac{x+11}{4x-3} > 0$$

	$(-\infty, -11)$		$(-11, \frac{3}{4})$		$(\frac{3}{4}, \infty)$
$x+11$	$-$	0	$+$		$+$
$4x-3$	$-$		$-$	0	$+$
$\frac{x+11}{4x-3}$	$+$	0	$-$	N.D.	$+$

$$x \in (-\infty, -11) \cup \left(\frac{3}{4}, \infty\right)$$

4.

$$\frac{-3x-2}{2x+7} > 1$$

- domácí

5.

$$\frac{3x-16}{(x+2)^2} > 1$$

$$\frac{3x-16}{(x+2)^2} - 1 > 0 \quad |\cdot(x+2)^2$$

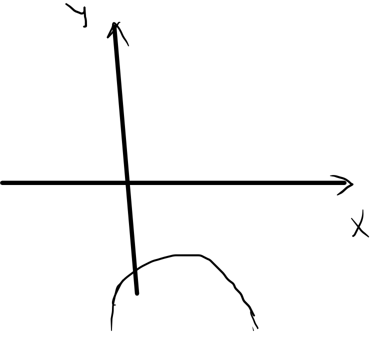
$$\frac{3x-16-(x^2+4x+4)}{(x+2)^2} > 0$$

$$\frac{3x-16-x^2-4x-4}{(x+2)^2} > 0$$

$$\frac{-x^2-x-20}{(x+2)^2} > 0 \qquad x \neq -2$$

$$-x^2-x-20 > 0$$

$$x_{1/2} = \frac{1 \pm \sqrt{1-4\cdot 20}}{2} \in \mathbb{C}$$



Znamo da je $-x^2-x-20<0$ za svaka $x\in\mathbb{R}$.

ovo nikad nije zadovoljeno, tj. nema rešenja nej-na

6.

$$\frac{-2x^2+x-1}{(2-x)(x+1)} < 1$$

$$\frac{-2x^2+x-1}{2x+2-x^2-x} - 1 < 0$$

$$\frac{-2x^2+x-1}{-x^2+x+2} - 1 < 0 \quad |\cdot(-x^2+x+2)$$

$$\frac{-2x^2+\cancel{x}-1+x^2-\cancel{x}-2}{-x^2+x+2} < 0$$

$$\frac{-x^2-3}{-x^2+x+2} < 0$$

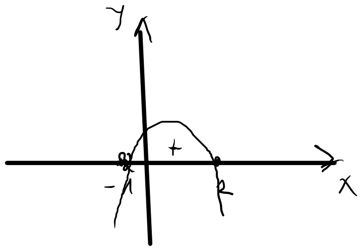
$$-(x^2+3) < 0 \quad \text{za svaka } x\in\mathbb{R}$$

$$\frac{-(x^2+3)}{-x^2+x+2} < 0$$

$$-x^2+x+2 > 0$$

$$x_{1/2} = \frac{-1\pm\sqrt{1+8}}{-2} = \frac{-1\pm 3}{-2}$$

$$x_1=-1 \qquad x_2=2$$



$x\in(-1,2)$

(7) $\dfrac{x^2-4}{-x+5} > 0$

$\dfrac{(x-2)(x+2)}{-x+5} > 0$

	$(-\infty, -2)$	$(-2, 2)$	$(2, 5)$	$(5, \infty)$
$x-2$	$-$	$-$ 0	$+$	$+$
$x+2$	$-$ 0	$+$	$+$	$+$
$-x+5$	$+$	$+$	$+$ 0	$-$
$\dfrac{x^2-4}{-x+5}$	$+$ 0	$-$ 0	$+$ N.D.	$-$

$x \in (-\infty, -2) \cup (2, 5)$

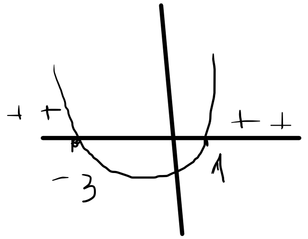
(8.) $\dfrac{x^2+4}{x^2+2x-3} > 0$

$x^2+4 > 0$ za svako $x \in \mathbb{R}$

$x^2+2x-3 > 0$

$x_{1/2} = \dfrac{-2 \pm \sqrt{4+12}}{2} = \dfrac{-2 \pm 4}{2}$

$x_1 = 1 \quad x_2 = -3$



$x \in (-\infty, -3) \cup (1, \infty)$

9.

$$\frac{x+1}{x-1} > \frac{1}{x+2}$$

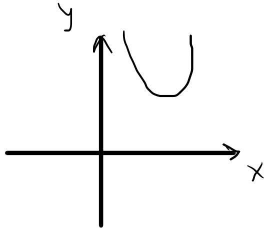
$$\frac{x+1}{x-1}\Big\backslash\cdot(x+2) - \frac{1}{x+2}\Big\backslash\cdot(x-1) > 0$$

$$\frac{x^2+2x+\cancel{x}+2-\cancel{x}+1}{(x-1)(x+2)} > 0$$

$$\frac{x^2+2x+3}{(x-1)(x+2)} > 0$$

$$x^2+2x+3=0$$

$$x_{1/2} = \frac{-2\pm\sqrt{4-12}}{2} \in \mathbb{C}$$



$\Rightarrow x^2+2x+3>0$ za svaki $x\in\mathbb{R}$

$$(x-1)(x+2)>0$$

	$(-\infty,-2)$	$(-2,1)$	$(1,\infty)$
$x-1$	$-$	$-$ 0	$+$
$x+2$	$-$ 0	$+$	$+$
$(x-1)(x+2)$	$+$	$-$	$+$

$x\in(-\infty,-2)\cup(1,\infty)$

10.

$$\frac{x^2-1}{2x-1} \geq 1$$

domaći

(11) $-7(13x-5)(3-11x) \leq 0$

$$\frac{5}{13} \;\Big|\cdot 11 \quad > \quad \frac{3}{11}\;\Big|\cdot 13$$

	$\left(-\infty, \frac{3}{11}\right)$	$\left(\frac{3}{11}, \frac{5}{13}\right)$	$\left(\frac{5}{13}, \infty\right)$
-7	$-$	$-$	$-$
$13x-5$	$-$	$-$ $\quad 0$	$+$
$3-11x$	$+$ $\quad 0$	$-$	$-$
$-7(13x-5)(3-11x)$	$+$ $\quad 0$	$-$ $\quad 0$	$+$

$$x \in \left[\frac{3}{11}, \frac{5}{13}\right]$$

(12) $\dfrac{-2x^2+3x-1}{x^2+x+1} > 0$

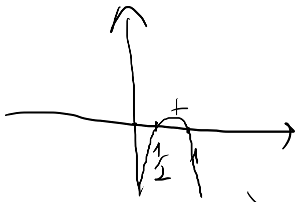
$-2x^2+3x-1=0$

$$x_{1/2} = \frac{-3 \pm \sqrt{9-8}}{-4} = \frac{-3 \pm 1}{-4}$$

$$x_1 = \frac{-2}{-4} = \frac{1}{2} \qquad x_2 = 1$$

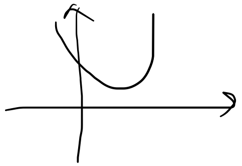
$x^2+x+1=0$

$$x_{1/2} = \frac{-1 \pm \sqrt{1-4}}{2} \in \mathbb{C}$$



$-2x^2+3x-1>0$

$$x \in \left(\frac{1}{2}, 1\right)$$



$\Downarrow$

$x^2+x+1>0 \quad \forall x \in \mathbb{R}$

13.

$$\frac{3x^2-17x+18}{x^2-5x+4} \le 2$$

$$\frac{3x^2-17x+18}{x^2-5x+4} - 2 \le 0 \quad / \cdot (x^2-5x+4)$$

$$\frac{3x^2-17x+18-2x^2+10x-8}{x^2-5x+4} \le 0$$

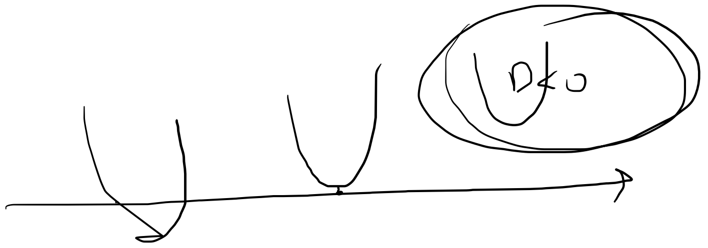
$$\frac{x^2-7x+10}{x^2-5x+4} \le 0$$

$$\frac{(x-5)(x-2)}{(x-4)(x-1)} \le 0$$

	$(-\infty,1)$	$(1,2)$	$(2,4)$	$(4,5)$	$(5,\infty)$
$x-5$	$-$	$-$	$-$	$-$ 0	$+$
$x-2$	$-$	$-$ 0	$+$	$+$	$+$
$x-4$	$-$	$-$	$-$ 0	$+$	$+$
$x-1$	$-$ 0	$+$	$+$	$+$	$+$
$\frac{(x-5)(x-2)}{(x-4)(x-1)}$	$+$ $\not\exists$	$-$ 0	$+$ $\not\exists$	$-$ 0	$+$

$x \in (1,2] \cup (4,5]$

14. Odrediti m tako da funkcija $y = x^2 - 2(m-1)x + m + 5$ bude stalno pozitivna.



$a = 1$

$b = -2(m-1)$

$c = m+5$

$D < 0$

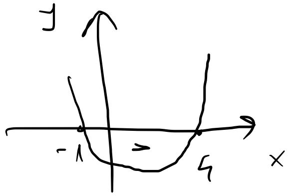
$$D = b^2 - 4ac = (-2(m-1))^2 - 4 \cdot 1 \cdot (m+5) =$$

$$= 4(m^2-2m+1) - 4m - 20 =$$

$$= 4(m^2-2m+1-m-5) = 4(m^2-3m-4) < 0$$

$m^2-3m-4=0$

$$m_{1/2} = \frac{3 \pm \sqrt{9+16}}{2} = \frac{3 \pm 5}{2} \qquad m_1 = 4 \quad m_2 = -1$$



$m \in (-1, 4)$

Sistemi jednacina

1.

$$2x - 3y = 11 \quad | \cdot (-3)$$
$$3x - 2y = 9 \quad | \cdot 2$$

$$5y = -15$$
$$y = -3$$

$$3x - 2(-3) = 9$$
$$3x + 6 = 9$$
$$3x = 3$$
$$x = 1$$

$$(1, -3)$$

2.

$$x - y = 3$$
$$5x^2 + y^2 = 45$$

$$x = 3 + y$$

$$5(3+y)^2 + y^2 = 45$$
$$5(9 + 6y + y^2) + y^2 = 45$$
$$\cancel{45} + 30y + 5y^2 + y^2 = \cancel{45}$$
$$6y^2 + 30y = 0$$
$$6y(y+5) = 0$$
$$y_1 = 0 \qquad y_2 = -5$$
$$x_1 = 3 \qquad x_2 = 3 - 5 = -2$$

Rešenja su $(3, 0)$, $(-2, -5)$

3.

$$x + y = -2$$
$$x + z = 1 \Rightarrow x = 1 - z$$
$$y + z = 1 \Rightarrow y = 1 - z$$

$$1 - z + 1 - z = -2$$
$$2 - 2z = -2$$
$$-2z = -4$$
$$z = 2$$

$$x = 1 - 2 = -1$$
$$y = 1 - 2 = -1$$

$$(-1, -1, 2)$$

4.

$$x^2+y^2=101$$
$$xy=10$$

$$x=\frac{10}{y}$$
$$\left(\frac{10}{y}\right)^2+y^2=101$$

$$x=\frac{10}{y}$$
$$\frac{100}{y^2}+y^2=101 \quad \Big/ \cdot y^2$$
$$100+y^4=101y^2$$
$$y^4-101y^2+100=0 \quad \text{bikvadratna j-na}$$

smena: $y^2=t$

$$t^2-101t+100=0$$
$$(t-100)(t-1)=0$$
$$t_1=100 \qquad t_2=1$$
$$y^2=100 \qquad y^2=1$$
$$y_{1/2}=\pm 10 \qquad y_{3/4}=\pm 1$$
$$x_{1/2}=\pm 1 \qquad x_{3/4}=\pm 10$$

$(1,10),(-1,-10),(10,1),(-10,-1)$

5.

$$x^2+y^2=74$$
$$x-y=2$$

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domaći..

6.
$$x^2-y^2=32$$
$$x:y=3$$
$$x=3y$$
$$(3y)^2-y^2=32$$
$$9y^2-y^2=32$$
$$8y^2=32$$
$$y^2=4$$
$$y_{1/2}=\pm 2$$
$$x_{1/2}=\pm 6$$
$$(6,2),(-6,-2)$$

7.
$$xy=12$$
$$x-2y=2$$
$$x=2+2y$$
$$(2+2y)y=12$$
$$2y+2y^2=12 \quad |:2$$
$$y+y^2=6$$
$$y^2+y-6=0$$
$$(y+3)(y-2)=0$$
$$y_1=-3 \quad y_2=2$$
$$x_1=2-6=-4 \quad x_2=2+4=6$$
$$(-4,-3),(6,2)$$

8.
$$\frac{1}{x}+\frac{2}{y}=3 \quad |\cdot(-1)$$
$$\frac{1}{x}-\frac{3}{y}=2 \quad (+)$$

I način:
$$-\frac{5}{y}=-1$$
$$y=5$$
$$\frac{1}{x}+\frac{2}{5}=3$$
$$\frac{1}{x}=\frac{3}{1}-\frac{2}{5}$$
$$\frac{1}{x}=\frac{13}{5}$$
$$x=\frac{5}{13}$$
$$\left(\frac{5}{13},5\right)$$

II način:
smena: $\frac{1}{x}=t$
$$\frac{1}{y}=p$$
$$t+2p=3 \quad |\cdot(-1)$$
$$t-3p=2$$
$$-5p=-1$$
$$p=\frac{1}{5}$$
$$\frac{1}{y}=\frac{1}{5} \Rightarrow y=5$$
$$t-\frac{3}{5}=2$$
$$t=\frac{2}{1}+\frac{3}{5}=\frac{13}{5}$$
$$\frac{1}{x}=\frac{13}{5}$$
$$x=\frac{5}{13}$$

a, b su parametri

(9)

$$ax + by = a + 2b$$
$$5x - ay = 5 - 2a$$

$$x = \frac{5 - 2a + ay}{5}$$

$$a \cdot \left(\frac{5-2a+ay}{5}\right) + by = a + 2b \quad | \cdot 5$$

$$\cancel{5a} - 2a^2 + a^2y + 5by = \cancel{5a} + 10b$$

$$y(a^2 + 5b) = 2a^2 + 10b$$

$$y(a^2 + 5b) = 2(a^2 + 5b)$$

$$y = \frac{2(\cancel{a^2+5b})}{\cancel{a^2+5b}}, \quad a^2 + 5b \neq 0$$

$$y = 2$$

$$x = \frac{5 - \cancel{2a} + \cancel{2a}}{5} = \frac{5}{5} = 1$$

Rez. $(1,2)$ p.t. valorile $a^2+5b \neq 0$

(10)

$$\frac{x}{a} + \frac{y}{b} = \frac{a(a^2+b^2)}{b(a+b)} \quad | \cdot \left(\frac{1}{a-b}\right)$$

$$\frac{y}{a-b} - \frac{3x}{a+b} = \frac{(a-b)^2}{a+b} \quad | \cdot \left(-\frac{1}{b}\right) \quad \downarrow (+)$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$a, b \neq 0$
$a - b \neq 0$
$a + b \neq 0$

$$\frac{x}{a(a-b)} + \frac{3x}{(a+b)b} = \frac{a(a^2+b^2)}{b(a+b)(a-b)} - \frac{(a-b)^2}{(a+b)b} \quad | \cdot (a-b)$$

$$x\left(\frac{1^{\,| \cdot b(a+b)}}{a(a-b)} + \frac{3^{\,| \cdot a(a-b)}}{(a+b)b}\right) = \frac{a^3 + ab^2 - (a-b)^3}{b(a+b)(a-b)}$$

$$x \frac{ab + b^2 + 3a^2 - 3ab}{a\cancel{b}(\cancel{a-b})(\cancel{a+b})} = \frac{\cancel{a^3} + ab^2 - \cancel{a^3} + 3a^2b - 3ab^2 + b^3}{\cancel{b}(\cancel{a+b})(\cancel{a-b})}$$

$$x \cdot \frac{3a^2 + b^2 - 2ab}{a} = 3a^2b - 2ab^2 + b^3$$

$$x \cdot \frac{(\cancel{3a^2+b^2-2ab})}{a} = b(\cancel{3a^2 - 2ab + b^2}), \quad 3a^2 - 2ab + b^2 \neq 0$$

$$\boxed{x = ab}$$

$$\frac{\cancel{a}b}{\cancel{a}} + \frac{y}{b} = \frac{a(a^2+b^2)}{b(a+b)} \Rightarrow \frac{y}{\cancel{b}} = \frac{a(a^2+b^2)}{b(a+b)} - b = \frac{a^3 + \cancel{ab^2} - \cancel{ab^2} - b^3}{\cancel{b}(a+b)}$$

$$\boxed{y = \frac{a^3 - b^3}{a+b} = \frac{(a-b)(a^2+ab+b^2)}{a+b}}$$

11. $a(x+y)+b(x-y)=2ab$
$a(x-y)+b(x+y)=a^2+b^2$

smena: $x+y=p \qquad x-y=t$

$ap+bt=2ab$
$at+bp=a^2+b^2$

$ap+bt=2ab \quad |\cdot b$
$bp+at=a^2+b^2 \quad |\cdot(-a) \quad \downarrow(+)$

$t(b^2-a^2)=2ab^2-a^3-ab^2$

$t(b-a)(b+a)=ab^2-a^3$

$t(b-a)(b+a)=a(b^2-a^2)$

$t\cancel{(b-a)}\cancel{(b+a)}=a\cancel{(b-a)}\cancel{(b+a)}$

$t=a$

$b-a\neq 0$
$b+a\neq 0$

$ap+ba=2ab$

$ap=ab \quad , a\neq 0$

$p=b$

$t=a \qquad x-y=a$
$p=b \qquad x+y=b$

$x-y=a$
$x+y=b \quad \downarrow(+)$

$2x=a+b$

$\boxed{x=\frac{a+b}{2}}$

$\frac{a+b}{2}-y=a$

$\frac{a+b}{2}-\frac{a}{1}=y$

$\frac{a+b-2a}{2}=y$

$\boxed{y=\frac{b-a}{2}}$

12. $\frac{b}{a}x-\frac{b}{c}y=bc-d \quad |\cdot\frac{c}{d} \quad \downarrow(+)$

$\frac{a}{c}x+\frac{c}{d}y=a^2+\frac{c^2}{b} \quad |\cdot\frac{b}{c}, \quad abcd\neq 0$

$x\left(\frac{bc}{ad}+\frac{ab}{c^2}\right)=\frac{bc^2}{d}-\cancel{c}+\frac{a^2b}{c}+\cancel{c}$

$x\left(\frac{\cancel{bc^3+a^2bd}}{a\cancel{d}c^{\cancel{2}}}\right)=\frac{\cancel{bc^3+a^2bd}}{\cancel{cd}}, \qquad bc^3+a^2bd\neq 0$

$x\cdot\frac{1}{ac}=1 \Rightarrow \boxed{x=ac}$

$\frac{b}{\cancel{a}}\cdot\cancel{a}c-\frac{b}{c}y=bc-d$

$-\frac{b}{c}y=\cancel{bc}-d-\cancel{bc}$

$-\frac{b}{c}y=-d$

$\boxed{y=\frac{cd}{b}}$

13.

$$(a+b)x-(a-b)y=-a^2+4ab+3b^2 \quad |\cdot(-b)$$
$$bx-ay=2(b^2-a^2) \quad |\cdot(a+b)$$
$$\downarrow (+)$$

$$y\left(b(a-b)-a(a+b)\right)=a^2b-4ab^2-3b^3+2(ab^2+b^3-a^3-a^2b)$$

$$y(\cancel{ab}-b^2-a^2-\cancel{ab})=a^2b-4ab^2-3b^3+2ab^2+2b^3-2a^3-2a^2b$$

$$-y(a^2+b^2)=-2a^3-a^2b-2ab^2-b^3$$

$$-y(a^2+b^2)=-2a(a^2+b^2)-b(a^2+b^2)$$

$$-y\cancel{(a^2+b^2)}=\cancel{(a^2+b^2)}(-2a-b) \qquad a^2+b^2\neq 0$$

$$-y=-2a-b$$

$$\boxed{y=2a+b}$$

$$bx-a(2a+b)=2b^2-2a^2$$

$$bx=2b^2-\cancel{2a^2}+\cancel{2a^2}+ab$$

$$\cancel{b}x=\cancel{b}(2b+a) \qquad b\neq 0$$

$$\boxed{x=2b+a}$$

Jednacine

1. $(x^4 - 8x^2 - 20)(x^3 - 1) = 0$

2. $-2x^4 + 14x^2 - 24 = 0$

3. $(x^4 - 7x^2 + 10)(x^2 + x + 1) = 0$

4. $x^6 - 3x^3 + 2 = 0$

5. $(x^2 + x)^2 - 8(x^2 + x) + 12 = 0$

6. $3x^{-4} + 5x^{-2} - 2 = 0$

7. $(6x^2+x)^2-2(6x^2-7x)-3=0$

8. $\frac{x^2-3}{x-1}-2\frac{x-1}{x^2-3}+1=0$