KVADRATNA JEDNAČINA

$$ax^2 + bx + c = 0,\ a \neq 0 \quad x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$D = b^2 - 4ac$$

$D>0 \quad x_1 \neq x_2, \quad x_1, x_2 \in \mathbb{R}$ (realni brojevi)

$D=0 \quad x_1 = x_2, \quad x_1, x_2 \in \mathbb{R}$

$D<0 \quad x_1 = a+bi \quad a,b \in \mathbb{R}$
$\qquad\ \ x_2 = a-bi$

$$b, c = 0 \quad ax^2 = \cancel{0}\ /:a \Leftrightarrow x_{1,2} = 0.$$

$$c = 0 \quad ax^2 + bx = 0 \Rightarrow x(ax+b) = 0,\ x_1 = 0,\ x_2 = -\frac{b}{a}.$$

a

$$b = 0 \quad ax^2 + c = 0,\ a<0 \text{ ili } c<0 \ x^2 = -\frac{c}{a} \rightarrow x_{1,2} = \pm\sqrt{\frac{-c}{a}}.$$

$2x^2 + 5 \overset{?}{=} 0$

$x^2 \geq 0 \quad 2x^2+5>0$ u skupu $\mathbb{R}$

$-2x^2 + 5 = 0 \Rightarrow 2x^2 = 5 \Rightarrow x^2 = \frac{5}{2}$

$x_1 = \sqrt{\frac{5}{2}} \quad x_2 = -\frac{5}{2}$

Bikvadratna jednačina

$$\boxed{ax^4 + bx^2 + c = 0.}$$

Smena $x^2 = t \Rightarrow at^2 + bt + c = 0.$

Vijetove formule

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$$ax^2 + bx + c = 0 \Leftrightarrow a(x - x_1)(x - x_2) = 0.$$

$$a(x^2 - xx_2 - xx_1 + x_1x_2) = 0 \Rightarrow a(x^2 - x(x_1 + x_2) + x_1x_2) = 0$$

$$ax^2 - ax(x_1 + x_2) + a(x_1x_2) = 0 \Rightarrow -a(x_1 + x_2) = b, \quad x_1 + x_2 = -\frac{b}{a}$$

$$a(x_1 \cdot x_2) = c, \quad x_1 \cdot x_2 = \frac{c}{a}.$$

$$(a+b)^2 = a^2+2ab+b^2$$

$$(a-b)^2 = a^2-2ab+b^2$$

$$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$$

$$(a-b)^3 = a^3-3a^2b+3ab^2-b^3$$

$$a^2-b^2 = (a-b)(a+b)$$

$$a^3-b^3 = (a-b)(\underbrace{a^2+ab+b^2}_{\substack{\vee \\ 0}})$$

$$a^3+b^3 = (a+b)(a^2-ab+b^2)$$

$$a^4-b^4 = (a^2)^2-(b^2)^2 = (a^2-b^2)\cdot(a^2+b^2)$$
$$= (a-b)(a+b)(\underbrace{a^2+b^2}_{\substack{\vee \\ 0}})$$

$$a^2+b^2 = \underbrace{a^2+2ab+b^2}-2ab$$
$$= (a+b)^2-(\sqrt{2ab})^2$$
$$= (a+b-\sqrt{2ab})(a+b+\sqrt{2ab})$$

1. Rešiti kvadratnu jednačinu

a) $\boxed{2x^2 + 3x - 5 = 0.}$ $\Rightarrow x_{1,2} = \frac{-3 \pm \sqrt{3^2 - 4\cdot 2\cdot(-5)}}{4} = \frac{-3 \pm \sqrt{9+40}}{4} = \frac{-3 \pm 7}{4}$

$x_1 = -\frac{10}{4} = -\frac{5}{2}$

$x_2 = 1$

$\Downarrow$

$2\left(x - \left(-\frac{5}{2}\right)\right)\cdot(x-1) = 0$

$2\left(x + \frac{5}{2}\right)(x-1) = 0 \Rightarrow (2x+5)(x-1) = 0$

b) $\boxed{\underset{a}{3}x^2 \underbrace{-(9k+2)}_{b}x + \underset{c}{6k} = 0.}$

$x_{1,2} = \frac{9k+2 \pm \sqrt{(-(9k+2))^2 - 4\cdot 3\cdot 6k}}{6}$ $\quad (-(a+b))^2 = (a+b)^2$

$x_{1,2} = \frac{9k+2 \pm \sqrt{81k^2 + 36k + 4 - 72k}}{6} = \frac{9k+2 \pm \sqrt{81k^2 - 36k + 4}}{6} = \frac{9k+2 \pm \sqrt{(9k-2)^2}}{6}$

$x_{1,2} = \frac{9k+2 \pm (9k-2)}{6}$

$x_1 = \frac{9k+2-(9k-2)}{6} = \frac{4}{6} = \frac{2}{3}$

$x_2 = \frac{9k+\cancel{2}+9k-\cancel{2}}{6} = \frac{18k}{6} = 3k$

$3\left(x - \frac{2}{3}\right)(x-3k) = 0$

$\Downarrow$

$(3x-2)(x-3k) = 0$

c) $x^4 - 10x^2 + 9 = 0.$ $\quad x^2 = t$

$t^2 - 10t + 9 = 0 \quad t_{1,2} = \frac{10 \pm \sqrt{10^2 - 4\cdot 1\cdot 9}}{2} = \frac{10 \pm \sqrt{64}}{2}$

$t_{1,2} = \frac{10 \pm 8}{2} \quad t_1 = 1 \quad x^2 = 1 \Rightarrow x_1 = 1,\ x_2 = -1$

$t_2 = 9 \quad x^2 = 9 \Rightarrow x_3 = 3,\ x_4 = -3$

$1\cdot(x-1)\cdot(x-(-1))\cdot(x-3)\cdot(x-(-3)) = 0 \Rightarrow (x-1)(x+1)(x-3)(x+3) = 0$

d) $x^6 - 5x^3 + 4 = 0.$ $\quad x^3 = t$

$t^2 - 5t + 4 = 0 \Rightarrow t_{1,2} = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5 \pm 3}{2}$

$t_1 = 1 \Rightarrow x^3 = 1 \Rightarrow x^3 - 1^3 = 0 \Rightarrow (x-1)(x^2 + x + 1^2) = 0$

$\Downarrow$

$x - 1 = 0$

$\boxed{x_1 = 1}$

$x^2 + x + 1 = 0 \Rightarrow x_{2,3} = \frac{-1 \pm \sqrt{1-4}}{2}$

$x_{2,3} = \frac{-1 \pm \sqrt{-3}}{2}$

$x_{2,3} = \frac{-1 \pm \sqrt{3\cdot(-1)}}{2}$

$i^2 = -1$

$x_{2,3} = \frac{-1 \pm \sqrt{3i^2}}{2}$

$\boxed{x_{2,3} = \frac{-1 \pm i\sqrt{3}}{2}}$

$t_2 = 4$

$\Downarrow$

$x^3 = 4 \Rightarrow x^3 - (\sqrt[3]{4})^3 = 0$

$(x - \sqrt[3]{4})(\underbrace{x^2 + x\sqrt[3]{4} + \sqrt[3]{4^2}}_{>0}) = 0$

$\Downarrow$

$\boxed{x = \sqrt[3]{4}}$ jedino realno rešenje

2. U jednačini $x^2 - 8x + q = 0$ odrediti q tako da jedan koren (jedno rešenje) jednačine bude tri puta veći od drugog.

$x_1, x_2 \rightarrow$ koreni (rešenja) jedn

$x_1 = 3x_2$

Vijetove formule

$a = 1$

$b = -8$

$c = q$

$x_1 + x_2 = -\frac{b}{a} \Rightarrow x_1 + x_2 = \frac{8}{1} = 8 \Rightarrow 3x_2 + x_2 = 8 \Rightarrow 4x_2 = 8 \Rightarrow x_2 = 2$

$x_1 = 3 \cdot 2 = 6$

$x_1 \cdot x_2 = \frac{c}{a} \Rightarrow x_1 \cdot x_2 = \frac{q}{1} = q \Rightarrow 6 \cdot 2 = q \Rightarrow q = 12$

$x^2 - 8x + 12 = 0$

$1 \cdot (x-6)(x-2) = 0$

3. Odrediti m tako da zbir kvadrata korena jednačine $x^2-(m+1)x+m=0$ bude jednak 10.

x_1, x_2 - KORENI

$$\boxed{x_1^2+x_2^2=10}$$

$a=1$

$b=-(m+1)$

$c=m$

$$x_1+x_2=-\frac{b}{a} \Rightarrow x_1+x_2=\frac{m+1}{1} \Big/^2 \Rightarrow (x_1+x_2)^2=(m+1)^2$$

$$x_1\cdot x_2=\frac{c}{a} \Rightarrow x_1\cdot x_2=\frac{m}{1}$$

$$\Downarrow$$

$$x_1^2+2x_1x_2+x_2^2=m^2+2m\cdot 1+1^2$$

$$10+\cancel{2m}=m^2+\cancel{2m}+1$$

$$10=m^2+1 \Rightarrow m^2=9$$

$$\Downarrow \qquad \Downarrow$$

$$m_1=3 \qquad m_2=-3$$

4. Sastaviti kvadratnu jednačinu čiji su koreni $-\frac{5}{6}$ i $\frac{7}{3}$.

$x_1 = -\frac{5}{6}$

$x_2 = \frac{7}{3}$

$$\underset{\overset{\parallel}{1}}{a}(x-x_1)(x-x_2)=0$$

$$\Downarrow$$

$$\left(x-\left(-\frac{5}{6}\right)\right)\left(x-\frac{7}{3}\right)=0$$

$$\left(x+\frac{5}{6}\right)\left(x-\frac{7}{3}\right)=0 \Rightarrow x^2-\frac{7}{3}x+\frac{5}{6}x-\frac{35}{18}=0$$

$$x^2+\frac{-7\cdot 2x+5x}{6}-\frac{35}{18}=0$$

$$x^2-\frac{9x}{6}-\frac{35}{18}=0 \Rightarrow \boxed{x^2-\frac{3}{2}x-\frac{35}{18}=0}$$

II Paskalov trougao

$(a+b)^n$, $(a-b)^n$

$n \in \mathbb{N}$
$\downarrow$
prirodan broj

1 $\Rightarrow (a+b)^0 = 1$

1 1 $\Rightarrow (a\pm b)^1 = 1a \pm b$

1 2 1 $\Rightarrow (a\pm b)^2 = a^2 \pm 2a^1b^1 + b^2$

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1 $\Rightarrow (a\pm b)^6 = a^6 \pm 6a^5b + 15a^4b^2 \pm 20a^3b^3 + 15a^2b^4 \pm 6ab^5 + b^6$

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

. . . .

III Binomna formula

$$(a+b)^n = \binom{n}{0} a^n \overset{=1}{b^0} + \binom{n}{1} a^{n-1}b^1 + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{k} a^{n-k}b^k + \ldots + \binom{n}{n} \overset{=1}{a^0} b^n$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \binom{n}{n-k}$$

$n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 1$

$5! = 5 \cdot \underbrace{4 \cdot 3 \cdot 2 \cdot 1}_{4!} \Rightarrow 5! = 5 \cdot 4!$

$0! \overset{def}{=} 1$

$1! = 1$

$$\binom{n}{0} = \frac{n!}{0!(n-0)!} = \frac{\cancel{n!}}{1 \cdot \cancel{n!}} = 1 \qquad \binom{n}{n} = \frac{n!}{n!(n-n)!} = \frac{\cancel{n!}}{\cancel{n!} \cdot \underset{1}{\underset{\shortparallel}{0!}}} = 1$$

a) Koristeći Paskalov trougao i binomnu formulu izračunati

$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

$$\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
\to 1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}$$

Binomnim → sami

$$(a-b)^6 = \underset{\substack{\shortparallel \\ 1}}{\binom{6}{0}} a^6 - \underset{\substack{\shortparallel \\ 6}}{\binom{6}{1}} a^5b + \underset{\substack{\shortparallel \\ 15}}{\binom{6}{2}} a^4b^2 - \underset{\substack{\shortparallel \\ 20}}{\binom{6}{3}} a^3b^3 + \underset{\substack{\shortparallel \\ 15}}{\binom{6}{4}} a^2b^4 - \underset{\substack{\shortparallel \\ 6}}{\binom{6}{5}} ab^5 + \underset{\substack{\shortparallel \\ 1}}{\binom{6}{6}} b^6$$

$$\binom{6}{1} = \frac{6!}{1!(6-1)!} = \frac{6!}{1\cdot 5!} = \frac{6\cdot\cancel{5!}}{\cancel{5!}} = 6$$

$$\binom{6}{5} = \frac{6!}{5!(6-5)!} = \frac{6!}{5!\,1!} = 6$$

$$\binom{6}{2} = \frac{6!}{2!(6-2)!} = \frac{6!}{2\cdot 1\cdot 4!} = \frac{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{2\cdot\cancel{4!}} = \frac{\cancel{6}\cdot 5}{\cancel{2}} = 15$$

$$\binom{6}{3} = \frac{6!}{3!\,3!} = \frac{\cancel{6}\cdot 5\cdot 4\cdot\cancel{3!}}{\cancel{3!}\,\cancel{3\cdot 2}\cdot 1} = 20$$

IV Srediti algebarske izraze

a) $\left(\frac{x}{x+y}+\frac{1}{x-y}+\frac{x+y-y^2}{y^2-x^2}\right)\cdot\frac{x^2+2xy+y^2}{x^3+y^3}=$.............................$[\frac{1}{x-y}]$

$$\left(\frac{x}{x+y}+\frac{1}{x-y}+\frac{x+y-y^2}{(y-x)(x+y)}\right)\cdot\frac{(x+y)^{\cancel{2}}}{(\cancel{x+y})(\underbrace{x^2-xy+y^2}_{>0})}=$$

$y-x=-x+y$
$=-(x-y)$

$$=\left(\frac{x}{x+y}+\frac{1}{x-y}+\frac{x+y-y^2}{-(x-y)(x+y)}\right)\cdot\frac{x+y}{x^2-xy+y^2}$$

USLOVI:

$x+y\neq 0 \Rightarrow x\neq -y$
$x-y\neq 0 \Rightarrow x\neq y$

$$=\left(\frac{x}{x+y}+\frac{1}{x-y}-\frac{x+y-y^2}{(x-y)(x+y)}\right)\cdot\frac{x+y}{x^2-xy+y^2}$$

$$=\frac{x\cdot(x-y)+1\cdot(x+y)-(x+y-y^2)}{(\cancel{x+y})(x-y)}\cdot\frac{\cancel{x+y}}{x^2-xy+y^2}$$

$$=\frac{x^2-xy+\cancel{x}+\cancel{y}-\cancel{x}-\cancel{y}+y^2}{x-y}\cdot\frac{1}{x^2-xy+y^2}=\frac{\cancel{x^2-xy+y^2}}{x-y}\cdot\frac{1}{\cancel{x^2-xy+y^2}}$$

$$=\frac{1}{x-y}$$

b) $\dfrac{1}{a(a-b)(a-c)} - \dfrac{1}{b(c-b)(b-a)} + \dfrac{1}{c(c-a)(c-b)} = \ldots\ldots\ldots\ldots\ldots\ldots \left[\dfrac{1}{abc}\right]$

$b-a = -(a-b)$, $c-a = -(a-c)$

$$\frac{1}{a(a-b)(a-c)} + \frac{1}{b(c-b)(a-b)} - \frac{1}{c(a-c)(c-b)} =$$

$$= \frac{b\cdot c(c-b) + ac(a-c) - ab(a-b)}{abc(a-b)(a-c)(c-b)} = \frac{bc^2 - b^2c + a^2c - ac^2 - a^2b + ab^2}{abc(a-b)(a-c)(c-b)}$$

$$= \frac{c^2(b-a) - c(b^2-a^2) + ab(b-a)}{abc(a-b)(a-c)(c-b)} = \frac{c^2(b-a) - c(b-a)(b+a) + ab(b-a)}{abc(a-b)(a-c)(c-b)}$$

$$= \frac{(b-a)(c^2 - c(b+a) + ab)}{abc(a-b)(a-c)(c-b)} = \frac{-\cancel{(a-b)}(c^2 - cb - ca + ab)}{abc\cancel{(a-b)}(a-c)(c-b)} = \frac{-c(c-b) - a(c-b)}{abc(a-c)(c-b)}$$

$$= \frac{-\cancel{(c-b)}(c-a)}{abc(a-c)\cancel{(c-b)}} = \ominus\frac{\ominus\cancel{(a-c)}}{abc\cancel{(a-c)}} = +\frac{1}{abc} = \boxed{\frac{1}{abc}}$$

Uslovi:

$a \neq 0$
$b \neq 0$
$c \neq 0$
$a-b \neq 0 \Rightarrow a \neq b$
$a-c \neq 0 \Rightarrow a \neq c$
$c-b \neq 0 \Rightarrow c \neq b$

$$\text{c) } \left(\frac{a}{a^2+a-6}+\frac{1}{a^3+3a^2-4a-12}-\frac{4}{6-a-a^2}\right)\cdot\left(\frac{a}{1}-\frac{4+3a}{a+3}\right)=\text{.....................}[1]$$

$$\left(\frac{a}{(a+3)(a-2)}+\frac{1}{a^2(a+3)-4(a+3)}-\frac{4}{-(a^2+a-6)}\right)\cdot\frac{a(a+3)-(4+3a)}{a+3}$$

$$=\left(\frac{a}{(a+3)(a-2)}+\frac{1}{(a+3)(a^2-2^2)}+\frac{4}{(a+3)(a-2)}\right)\cdot\frac{a^2+\cancel{3a}-4-\cancel{3a}}{a+3}$$

$$=\left(\frac{a}{(a+3)(a-2)}+\frac{1}{(a+3)(a-2)(a+2)}+\frac{4}{(a+3)(a-2)}\right)\cdot\frac{(a-2)(a+2)}{a+3}$$

$$=\frac{a\cdot(a+2)+1+4\cdot(a+2)}{(a+3)\cancel{(a-2)}\cancel{(a+2)}}\cdot\frac{\cancel{(a-2)}\cancel{(a+2)}}{a+3}$$

$$=\frac{a^2+2a+1+4a+8}{(a+3)^2}=\frac{a^2+6a+9}{(a+3)^2}=\frac{\cancel{(a+3)^2}}{\cancel{(a+3)^2}}=\boxed{1}$$

$$a^2+a-6=0$$

$$a_{1,2}=\frac{-1\pm\sqrt{1^2-4\cdot1\cdot(-6)}}{2}$$

$$a_{1,2}=\frac{-1\pm5}{2}$$

$$a_1=-3 \quad a_2=2$$

Uslovi

$a+3\neq0 \Rightarrow a\neq-3$

$a-2\neq0 \Rightarrow a\neq2$

$a+2\neq0 \Rightarrow a\neq-2$

d) $\left(\frac{(a+b)^2+2b^2}{a^3-b^3}-\frac{1}{a-b}+\frac{a+b}{a^2+ab+b^2}\right)\cdot\left(\frac{1}{b}-\frac{1}{a}\right)=$.................... $\left[\frac{1}{ab}\right]$

$$\left(\frac{a^2+2ab+b^2+2b^2}{\underbrace{(a-b)(a^2+ab+b^2)}_{\neq 0}}-\frac{1}{a-b}+\frac{a+b}{a^2+ab+b^2}\right)\cdot\frac{a-b}{ba}$$

USLOVI

$a-b\neq 0 \Rightarrow a\neq b$

$b\neq 0$

$a\neq 0$

$$=\frac{a^2+2ab+3b^2-(a^2+ab+b^2)+(a+b)\cdot(a-b)}{(\cancel{a-b})(a^2+ab+b^2)}\cdot\frac{\cancel{a-b}}{ba}$$

$$=\frac{\cancel{a^2}+2ab+3b^2-\cancel{a^2}-ab-b^2+a^2-b^2}{(a^2+ab+b^2)\cdot ba}=\frac{\cancel{ab+b^2+a^2}}{\cancel{(a^2+ab+b^2)}\cdot ba}=\boxed{\frac{1}{ab}}$$

e) $\left(\frac{x^3+y^3}{x-y} \div \frac{x+y}{x-y}\right) \cdot \frac{(x+y)^3-x^3-y^3}{3x^2y+3xy^2} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots [x^2-xy+y^2]$

$$\left(\frac{\cancel{(x+y)}(x^2-xy+y^2)}{\cancel{x-y}} \cdot \frac{\cancel{x-y}}{\cancel{x+y}}\right) \cdot \frac{\cancel{x^3}+3x^2y+3xy^2+\cancel{y^3}-\cancel{x^3}-\cancel{y^3}}{3xy(x+y)}$$

$$= (x^2-xy+y^2) \cdot \frac{\cancel{3xy}\cancel{(x+y)}}{\cancel{3xy}\cancel{(x+y)}} = x^2-xy+y^2$$

USLOVI : $x-y \neq 0 \Rightarrow x \neq y$

$x+y \neq 0 \Rightarrow x \neq -y$

$x \neq 0$

$y \neq 0$

f) $\left(\frac{x-a}{1+ax} - \frac{x-b}{1+bx}\right) \div \left(\frac{1}{1} + \frac{(x-a)(x-b)}{(1+ax)(1+bx)}\right) = \ldots\ldots\ldots\ldots\ldots\ldots [\frac{b-a}{ab+1}]$

$$\frac{(x-a)\cdot(1+bx) - (x-b)(1+ax)}{(1+ax)(1+bx)} : \frac{(1+ax)(1+bx)+(x-a)(x-b)}{(1+ax)(1+bx)} =$$

$$= \frac{x+bx^2-a-ab\cancel{x}-(x+ax^2-b-ba\cancel{x})}{(1+ax)(1+bx)} : \frac{1+\cancel{bx}+\cancel{ax}+abx^2+x^2-\cancel{xb}-\cancel{ax}+ab}{(1+ax)(1+bx)}$$

$$= \frac{\cancel{x}+bx^2-a+\cancel{abx}-\cancel{x}-ax^2+b+\cancel{bax}}{\cancel{(1+ax)}\cancel{(1+bx)}} \cdot \frac{\cancel{(1+ax)}\cancel{(1+bx)}}{1+abx^2+x^2+ab}$$

$$= \frac{bx^2-a-ax^2+b}{1+abx^2+x^2+ab} = \frac{x^2(b-a)+(b-a)}{x^2(ab+1)+(ab+1)} = \frac{(b-a)\cancel{(x^2+1)}}{(ab+1)\cancel{(x^2+1)}}$$

0

$$= \boxed{\frac{b-a}{ab+1}}$$

USLOVI

$1+ax \neq 0 \Rightarrow x \neq -\frac{1}{a}$

$1+bx \neq 0 \Rightarrow x \neq -\frac{1}{b}$

$ab+1 \neq 0 \Rightarrow b \neq -\frac{1}{a}$